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Problem 1. Mating.

In certain species of birds, males can choose to be faithful (F) or philanderers (P); females can
choose to be coy (C) or loose (L). Coy females insist on a long courtship before copulating, while loose females do not. Faithful males tolerate a long courtship and help rear their young (sharing equally the cost of rearing), while philanderers do not wait and do not help. The value of having an o_spring is v (> 0) for the male and also v for the female. The total cost of rearing an o_spring is 2r > 0. The cost of prolonged courtship is w (> 0) for the male and also w for the female. Finally, we assume that v > r + w.

(a) Interpret the condition v > r + w.
(b) Write down the Normal form of the game.

(c) Find the set of (pure and mixed) Nash Equilibria if v > 2r. What happens as v increases? Interpret.

(d) Find the set of (pure and mixed) Nash equilibria if v < 2r. What happens as v increases? Interpret.
Problem 2. Public good provision
Two players have to simultaneously decide how much to contribute to the provision of a public good. If player 1 contributes x and player 2 contributes y, then the value of the public good is 2(x + y + xy), which they each receive. They can choose any non-negative number as their contribution.

The cost of contributing x for player 1 is x^2. The cost of contributing y for player 2 is ty^2, where the value of t is private information for player 2. Player 1 believes that t = 1 with probability p and t = 2 with probability 1 - p. This belief is common knowledge.

(a) What are the set of actions, types and beliefs of both players?
(b) Find the Bayesian Nash equilibrium of the game.
(c) Compare the quantities chosen by both players. Interpret (important).
(d) What happens as p -> 1?

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Problem 1.Mating

(a) The condition v > r + w implies that if courtship results in sharing of costs of raising the offspring, then it is worth it to both the male and female birds.

(b) The normal form of the game can be written as follows

Female

Coy Loose

Faithful

Male

Philanderer

(c) Since $v > 2r$ in this case,

The male's best responses to all of female's actions are $v - r - w$ (box (1)) and v (box (4)) and female's best responses to all of male's actions are $v - r$ (box (2)) and $v - 2r$ (box (4)). Since Nash equilibrium exists where male's best response is the same as female's best response, box (4), it exists where philandering males and loose females meet. This is the Nash equilibrium with respect to pure strategy.

In the case of mixed strategy, if p is the probability that a male will be faithful, $(1 - p)$ is the probability that it will be philandering. Similarly, if q is the probability that a female will be coy, $(1 - q)$ is the probability that it will be loose. For a given probability q chosen by female, the expected payoff for every male's pure strategy

$\pi_1 = (v - r - w)q + (v - r)(1 - q)$

$\pi_1 = 0 + v(1 - q)$

Equating both payoffs,

$(v - r - w)q + (v - r)(1 - q) = 0 + v(1 - q)$

$(v - r - w)q + (v - r) - (v - r)q = v - vq$

$-wq + v - r = v - vq$

$(v - w)q = r$

$q = r / (v - w)$

For a given probability p chosen by male, the expected payoff for every female's pure strategy

$\pi_2 = (v - r - w)p + 0$

$\pi_2 = (v - r)p + (v - 2r)(1 - p)$

Equating both payoffs,

$(v - r - w)p + 0 = (v - r)p + (v - 2r)(1 - p)$

$(v - r - w)p - (v - r)p = (v - 2r) - (v - 2r)p$

$-wp + (v - 2r)p = (v - 2r)$

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$(v - 2r - w)p = (v - 2r)$

$p = (v - 2r)/ (v - 2r - w)$

Hence the mixed strategy Nash equilibrium is $(p = (v - 2r)/ (v - 2r - w), q = r / (v - w))$. Hence when $v > 2r$, both pure strategy and mixed strategy Nash equilibriums exist. As v (the value of having an offspring) increases, there will not be a great difference since there is only one pure strategy Nash equilibrium. However, the probability of [philanderers, loose] being played in the mixed equilibrium decreases, as v increases.

(d) Since $v < 2r$ in this case,

The male's best responses to all of female's actions are $v - r - w$ (box (1)) and v (box (4)) and female's best responses to all of male's actions are $v - r$ (box (2)) and 0 (box (3)). Since Nash equilibrium exists where male's best response is the same as female's best response, box (4), there is no Nash equilibrium with respect to pure strategy.

In the case of mixed strategy, if p is the probability that a male will be faithful, $(1 - p)$ is the probability that it will be philandering. Similarly, if q is the probability that a female will be coy, $(1 - q)$ is the probability that it will be loose. For a given probability q chosen by female, the expected payoff for every male's pure strategy

$\pi_1 = (v - r - w)q + (v - r) (1 - q)$

$\pi_1 = 0 + v (1 - q)$

Equating both payoffs,

$(v - r - w)q + (v - r) (1 - q) = 0 + v (1 - q)$

$(v - r - w)q + (v - r) - (v - r)q = v - vq$

$-wq + v - r = v - vq$

$(v - w)q = r$

$q = r / (v - w)$

For a given probability p chosen by male, the expected payoff for every female's pure strategy

$\pi_2 = (v - r - w)p + 0$

$\pi_2 = (v - r) p + (v - 2r) (1 - p)$

Equating both payoffs,

$(v - r - w)p + 0 = (v - r) p + (v - 2r) (1 - p)$

$(v - r - w)p - (v - r) p = (v - 2r) - (v - 2r)p$

$-wp + (v - 2r)p = (v - 2r)$

$(v - 2r - w)p = (v - 2r)$

$p = (v – 2r)/ (v – 2r – w)$

But in this case, p will be negative, since $v < 2r$. Hence let us assume that p is the probability that a male will be philandering, $(1 – p)$ is the probability that it will be faithful. In this case,

$\pi_2 = 0 + (v – r – w)(1 – p)$

$\pi_2 = (v – 2r) p+ (v – r) (1 - p)$

Equating both payoffs,

$0 + (v – r – w)(1 – p) = (v – 2r) p+ (v – r) (1 - p)$

$-w + wp = (v – 2r) p$

$(v – 2r - w) p = -w$

$p = w/ (2r + w – v)$

Hence the mixed strategy Nash equilibrium is $(p = w/ (2r + w – v), q = r / (v - w))$. Hence when $v < 2r$, only mixed strategy Nash equilibriums exists and there is no pure strategy Nash equilibrium. As v (the value of having an offspring) increases, it gradually overtakes 2r and offers a pure strategy Nash equilibrium. The case will then be equal to the answer in (c).

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Problem 2. Public good provision

(a) The set of actions available to both players is that both can decide to contribute or not to contribute. There are four options, namely (1) player 1 contributes and player 2 does not contribute, (2) player 2 contributes and player 1 does not contribute, (3) both players do not contribute and (4) both players contribute. Thus with two actions for both players, we have four different situations.

The cost of contributing y for player 2 is ty^2, where t can be 1 or 2. This represents the types of player 2. If $t = 1$, it is low-cost type and this has probability p. If $t = 2$, it is high-cost type and this has probability $(1 – p)$. Thus the two types are low cost and high cost. Both players believe that the cost of their contribution is known publicly. Both players also believe that the probabilities of how the cost varies are also known to both.

(b) Baynesian Nash equilibrium

The payoff function of player 1 is

$u_1 = 2(x + y + xy) – x^2$

But since there are two types for player 2, the payoff function for player 1 is,

$u_1 = p [2(x + y_L + xy_L) – x^2] + (1 – p) [2(x + y_H + xy_H) – x^2]$

The first order derivative of the above with respect to x is

$u_1 = p [2(1 + y_L) – 2x] + (1 – p)[2(1 + y_H) – 2x]$

$u_1 = p[2 + 2y_L - 2x] + (1 - p)[2 + 2y_H - 2x]$

$u_1 = 2p + 2y_Lp - 2xp + 2 + 2y_H - 2x - 2p - 2y_Hp + 2xp$

$u_1 = 2y_Lp - 2y_Hp + 2 + 2y_H - 2x$

On equating the first derivative to zero,

$2y_Lp - 2y_Hp + 2 + 2y_H - 2x = 0$

$y_Lp - y_Hp + 1 + y_H - x = 0$

$x = y_Lp - y_Hp + 1 + y_H$

The second order derivative of this u_1 is -2, which is less than zero and hence it is satisfied.

The payoff function of player 2 is

$u_2 = 2(x + y + xy) - ty^2$

The first order derivative of the above with respect to y is

$u_2 = 2 + 2x - 2ty$

On equating the first derivative to zero,

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$2 + 2x - 2ty = 0$

$1 + x - ty = 0$

$y = (1 + x)/ t$

When $t = 1$, $y_L = 1 + x$

When $t = 2$, $y_H = (1 + x)/2$

The second order derivative of this u_2 is -2t, which is less than zero and hence it is satisfied.

On substituting the derived in x,

$x = (1 + x)p - ((1 + x)/2)p + 1 + (1 + x)/2$

$x = p + xp - p/2 - xp/2 + 1 + ½ + x/2$

$x/2 - xp/2 = p/2 + 3/2$

$x - xp = p + 3$

$x = (p +3)/ (1 - p)$

On substituting the derived in y,

$y_L = 1 + (p +3)/ (1 - p)$

$y_H = (1 + (p + 3) / (1 - p))/2$

(c) Assuming p = 0.5, x = 7, y_L = 8 and y_H = 4. Hence player 1 contributes quantity 7 to the provision of the public good and player 2 contributes quantity 8 when cost is low or 4 when cost is high to the provision of the public good. Hence with the cost unknown to any of the players, they end up paying higher with no payoff for them. When the cost is well known, they can contribute accordingly. Contribution is more attractive when the costs are lower. Besides, when this information is known, it is far better that both the players will contribute. In the current case, there are chances that they may not contribute and hence this can be eliminated.

(d) When p nears 1, t will be equal to one and there is only one type of player 2 (no low cost and high cost differentiation). Both the players will decide not to contribute, since they cooperate and decide on not contributing owing to any reason such as cost or other dependent factors.

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