Sample Probability and Statistics for Computer Science Assignment | ExpertsMind.com

Solutions:

COMP 233 Probability and Statistics for Computer Science

Question 1 A carton of 12 rechargeable batteries contains two batteries that are defective.

(a) In how many ways can an inspector choose three (3) of the batteries?

(b) In how many ways can an inspector choose three (3) of the batteries and get none of the defective batteries?

(c) In how many ways can an inspector choose three (3) of the batteries and get one of the defective batteries?

(d) In how many ways can an inspector choose three (3) of the batteries and get both of the defective batteries?

Solution:

Here we are given:

$n = 12$

x = Number of defective batteries = 2.

a) The three batteries can be chosen out of 12 batteries in

$C(12,3) = 220$.

b) Here we need to choose 3 batteries condition that no one is defective.

there are 10 non defective batteries and 2 defective batteries.

Hence the required number of ways = $C(10,3)*C(2,0) = 120*1=120$

c) Now we need to find the probability of getting one defective battery and two non defective batteries.

The number of ways to do this is = $C(10,2)*C(2,1) = 45*2 = 90$

d) Now we are supposed to find the probability of getting one non defective and two defective batteries.

The number of ways to do this are = C(10,1)*C(2,2) =10*1 = 10

Question2 A software engineer wants to move across the country from Vancouver to Montréal to work. On different days, she mails a cover letter and resume to three different companies in Montréal. Suppose a letter sent from Vancouver to Montréal has a probability of 0.75 of reaching Montréal within three (3) days.

(a) What is the probability that exactly two (2) of the three (3) letters will reach Montréal

within three (3) days?

(b) What is the probability that at least one (1) of the three (3) letters will reach Montréal within three (3) days?

(c) If the three (3) letters are mailed together at the same time and location, how does your conclusion in part (a) change?

Solution:

Given: P(The mail reaches within 3 days) = 0.75.

a) Exactly 2 of 3 letters reaches within 3 days

= P(Reaching)*P(Reaching)*P(Not reaching)

= 0.75*0.75*(1-0.75)

= 0.141

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b) Probability that at least one of the three letters reaches within 3 days

= P(one letter reaches) + P(two letters reaches)+P(3 letters reaches)

= $0.75*(1-0.75)^2+0.75^2*(1-0.75)+0.75^3$

= 0.6094

c) Here all the three letters are being sent at same time and same day. So we may consider it as a single mail and all those will reach at the same time.

Hence the event two of three mails reach is an impossible event.

Hence the required probability will be 0 in this case.

Question 3 Two software consulting firms V and W consider bidding on a large programming project, which may or may not be awarded depending on the amounts of the bids. Firm V submits a bid and the probability is 3/4 that it will get the project provided that firm W does not bid. The probability is 3/4 that W will bid, and if it does, the probability that V will get the project is 1/3.

(a) What is the probability that V will get the project?

(b) If V gets the project, what is the probability that W did not bid?

Solution:

Let A = V gets the project

B = W will bid.

Given: P(A/B') = 3/4 B': Complement of B (Not B).

P(B) = 3/4

P(A/B) = 1/3

a) P(A) = ?

Now we know by the Baye’s theorem.

P(A) = P(A/B)*P(B)+P(A/B')*P(B')

= 1/3 * 3/4 + 3/4 * (1- 1/4)

= 0.4375

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b) P(B'/A) = ?

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.25}{0.4375} = 0.5714$$

P(B’/A) = 1 – 0.5814 = 0.4285

Question 4 Two balls, each equally likely to be coloured either red or blue, are put in an urn. At each stage one of the balls is randomly chosen, its colour is noted, and it is then returned to the urn. If the first two balls chosen are coloured red, what is the probability that

(a) both balls in the urn are colored red;

(b) the next ball chosen will be red?

Solution:

Here the Simple random sampling with replacement is used, hence the two draws are independent of each other.

a) P(Red AND Red) = ½ * ½ = ¼ = 0.25

b) P(Red) = 1/5 = 0.5

Question 5 Suppose that 5 independent trials, each of which results in any of the outcomes 0, 1, or 2, with respective probabilities 0.3, 0.5, and 0.2, are performed. Find the probability that both outcome 1 and outcome 2 occur at least once. (*Hint*: Consider the complementary probability.)

Solution:

Here we are given the following discrete distribution:

X	P(x)
0	0.3
1	0.5
2	0.2

We are interested to find the probability that both outcome 1 and outcome 2 occur at least once.

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For one draw the probability that both outcome 1 and outcome 2 occur at least once

= [1 – P(X=1)]*[1-P(X=2)]

= (1 – 0.5)*(1-0.2) = 0.4

The five trials are independent. Hence for 5 trials the required probability = 0.4^5 = 0.01024

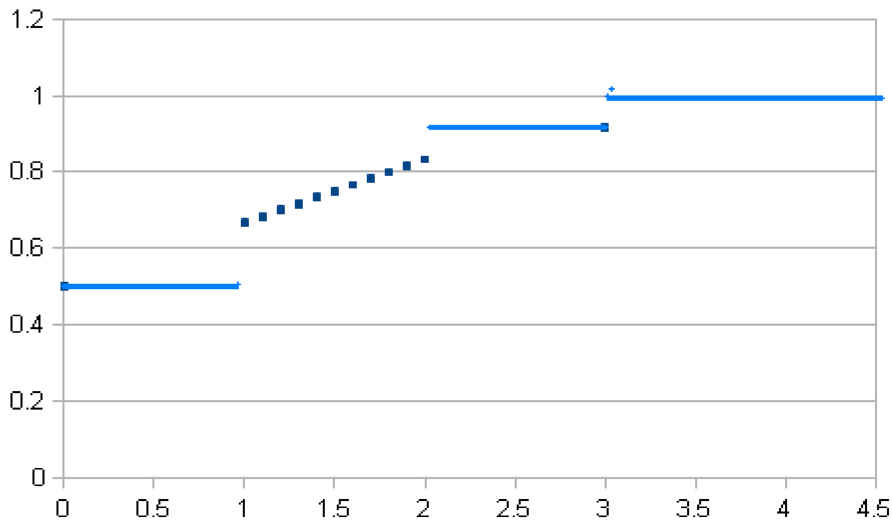
Question 6 The cumulative distribution function of the random variable X is given

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/2 & 0 \le x < 1 \\ 1/2 + (x/6) & 1 \le x < 2 \\ 11/12 & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

(a) Plot this cumulative distribution function.
(b) What is $P\{X > 1.5\}$?
(c) What is $P\{2 < X \le 4\}$?
(d) What is $P\{X < 3\}$?
(e) What is $P\{X = 2\}$?

Solution: Here we are given a mixed cumulative distribution function.

a) This function can be plotted as follows:



b) Now we will firstly find the pdf corresponding to this distribution function as follows:

$$f(x) = \begin{cases} \frac{1}{2} & if\ x = 0 \\ \frac{x}{6} & if\ 1 \le x < 2 \\ 0.0833 & if\ x = 2 \\ 0 & otherwise \end{cases}$$

P(X>1.5) = 1 – P(X≤1.5)

= 1 – [P(X=0)+P(1≤X<1.5)]

$$= 1 - \left(\frac{1}{2} - \left[\frac{x^2}{2*6}\right]_{x=1}^{1.5}\right)$$

$$= 1 - \left(\frac{1}{2} - \left[\frac{1.5^2}{12} - \frac{1^2}{12}\right]\right)$$

=0.6042

c) P(2<X≤4) = 0
d) P(X≤3) = 1 ……… Using distribution function.
e) P(X = 2) = 0.0833

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Question 7 If the density function of X equals

$$f(x) = \begin{cases} ce^{-2x} & 0 < x < \infty \\ 0 & x < 0 \end{cases}$$

(a) For what value of c is this a density function?
(b) What is P{X > 1}?

Solution:

a) We know the property of probability density function *f(x)*:

$$\int f(x)dx = 1$$

We will use this fact to find the value of c.

$$\int f(x)dx = 1$$

i.e.

$$\int_0^{\infty} ce^{-2x}dx = 1$$

hence,

$$c * \int_0^{\infty} e^{-2x} dx = 1$$

So, c * ½ = 1 gives c=2

b) P(X>1) = $\int_1^{\infty} 2 * e^{-2x} dx = \frac{1}{e^2}$ = 0.1353

Question 8 The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} 6e^{(-2x-3y)} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the probability density function of X, $f_X(x)$.

(b) Compute the probability density function of Y, $f_Y(y)$.

(c) Are random variables X and Y independent?

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Solution:

a)

$$f_X(x) = \int f(x,y)dy = \int_0^{\infty} 6 * e^{-2x-3y} dy$$

$$f_X(x) = \begin{cases} 2e^{-2x} & \text{if } 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

b)

$$f_Y(y) = \int f(x,y)dx = \int_0^{\infty} 6 * e^{-2x-3y} dx$$

$$f_Y(y) = \begin{cases} 3e^{-3y} & \text{if } 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

c) We know that X and Y are independent if their joint distribution is the product of their marginal distributions.

i.e. if f(x)*f(y)=f(x, y)

$$\text{Consider}, f(x) * f(y) = 2e^{-2x} * 3e^{-3y}$$

$= 6 * e^{-2x-3y}$

$= f(x, y)$

Hence the variables are independent.