Exercise 3 - 40%

Two firms, i = 1, 2, sell differentiated goods. The firms set prices p_1 and p_2 simultaneously. We assume that prices have to be non-negative, $p_1, p_2 \ge 0$. The demand function facing firm *i* is given by

$$q_i\left(p_i, p_j\right) = 2 - p_i + p_j$$

(do not worry about negative demand when doing this exercise - in equilibrium demand will be positive).

Firm 1 can have low marginal cost, c_L , or high marginal cost, c_H , where $0 < c_L < c_H < 2$. Firm 1 knows whether its marginal cost is low or high. Firm 2 only knows that the probability that Firm 1 has high marginal cost is θ , where $0 < \theta < 1$. Firm 2's marginal cost is c = 1 and this is known by both firms. (All this is common knowledge for the firms).

- Formulate the situation as a static game of incomplete information, i.e., specify action spaces, type spaces, beliefs, and payoff functions for the two firms.
- 2. Find the best response functions.
- 3. Find the Bayesian Nash equilibrium of the game (i.e., find the equilibrium prices).
- 4. Compare the BNE prices if Firm 1's realized cost is high with the equilibrium prices in the game where it is common knowledge that Firm 1 has high marginal cost (c_H).
- 5. Suppose now that Firm 1 does not observe its own marginal cost before it sets its price. I.e., Firm 1 only knows that the probability that it has high marginal cost (c_H) is θ . Everything else is as in the original set-up. Which equilibrium concept would you use to solve this game? Explain.



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Answer# 1:

Given the demand function: $q_i (p_i, p_j) = 2 - p_i + p_j (i = 1, 2 \text{ and } i \neq j)$

or, $p_i = 2 + p_j - q_i \rightarrow p_1 = 2 + p_j - q_1$ and $p_2 = 2 + p_j - q_2$

$$\rightarrow$$
 R_i = Revenue of firm i = p_i q_i = 2q_i + p_j q_i - q_i²

Thus $R_1 = 2q_1 + p_j q_1 - q_1^2$ = Revenue of 1st firm,

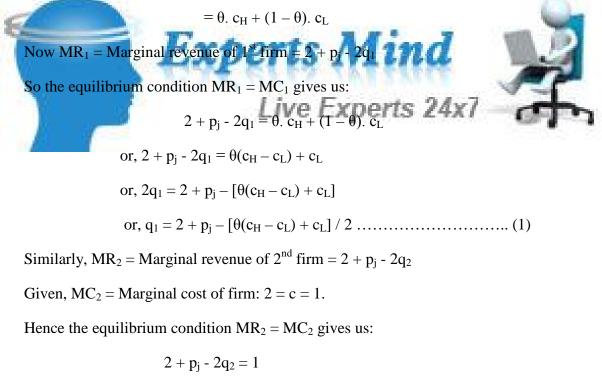
and $R_2 = 2q_2 + p_j q_2 - q_2^2 = Revenue of 2^{nd}$ firm.

According to the problem, firm: 2 only know that the probability of having high MC of firm: 1 is ' θ '.

So formally, $p(c_H) =$ Probability of having high MC = θ and

 $p(c_L) = Probability of having low MC = 1 - \theta$ where $\theta < 1$.

Hence we can express MC_1 = Marginal cost for firm: $1 = p(c_H)$. $c_H + p(c_L)$. c_L



or, $2q_2 = 1 + p_j$ or, $q_2 = 1 + p_j / 2$ (2)

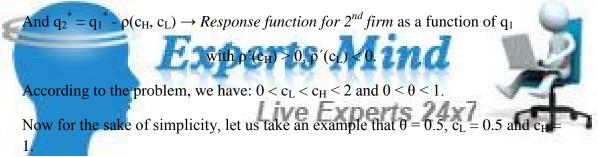
Equations: (1) and (2) show the situations of static game of incomplete information.

The payoff of 1^{st} firm: $S_1(\theta) = 0$ and the payoff of 2^{nd} firm: $S_2(\theta) = 1$.

Answer# 2:

According to equation: (1) we have:

$$\begin{aligned} q_{1} &= 2 + p_{j} - \left[\theta(c_{H} - c_{L}) + c_{L}\right] / 2 \\ \text{or, } q_{1} &= \left[(1 + p_{j}) + 1 - \theta(c_{H} - c_{L}) - c_{L}\right] / 2 \\ \text{or, } q_{1} &= \left[(1 + p_{j}) / 2\right] + \left[1 - \theta(c_{H} - c_{L}) - c_{L} / 2\right] \dots (3) \end{aligned}$$
We know that: $(1 + p_{j}) / 2 = q_{2}$. So substituting the value of q_{2} in equation: (3) we get:
 $q_{1}^{*} = q_{2}^{*} + \left[1 - \theta(c_{H} - c_{L}) - c_{L} / 2\right] = q_{2}^{*} + \rho(c_{H}, c_{L}) \dots (4)$
where $q_{1}^{*} =$ Equilibrium demand function for 1^{st} firm
 $= Response \ function \ for \ 1^{\text{st}} \ firm \ as \ a \ function \ of \ q_{2} \ with \ \rho'(c_{H}) > 0, \ \rho'(c_{L}) < 0. \end{aligned}$



 $\rho'(c_L) <$

Thus putting the values of c_L , c_H , and θ in equation: (4) we get:

 $q_1^* = q_2^* + [(1 - 0.25 - 0.5) / 2] = q_2^* + 0.125$ – which is a simplified response

function for firm: 1 with known values of the variables.

In a similar fashion, we can derive the simplified response function for the 2^{nd} firm with known values of the variables c_L , c_H , and θ .

However the two equilibrium values of q_1 and q_2 will become *equal* when

$$1 - \theta(\mathbf{c}_{\mathrm{H}} - \mathbf{c}_{\mathrm{L}}) - \mathbf{c}_{\mathrm{L}} / 2 = 0$$

or,
$$1 - \theta(\mathbf{c}_{\mathrm{H}} - \mathbf{c}_{\mathrm{L}}) - \mathbf{c}_{\mathrm{L}} = 0$$

or,
$$\theta(\mathbf{c}_{\mathrm{H}} - \mathbf{c}_{\mathrm{L}}) = 1 - \mathbf{c}_{\mathrm{L}} \rightarrow \mathbf{\theta} = (\mathbf{1} - \mathbf{c}_{\mathrm{L}}) / (\mathbf{c}_{\mathrm{H}} - \mathbf{c}_{\mathrm{L}})$$

In other words, $q_1^* = q_2^*$ when $\theta = (1 - c_L) / (c_H - c_L)$ – which contemplates the situation of a static game with complete information about the market (Pure Strategy Nash Equilibrium solution).

Answer# 3:

From our previous discussion, we have:

$$p_1 = 2 + p_j - q_1$$
 and $p_2 = 2 + p_j - q_2$

Therefore, $p_1=2+p_j$ - $\left[2+p_j-\left\{\theta(c_H-c_L)+c_L\right\}\right]/2$

as
$$q_1 = [2 + p_j - \{\theta(c_H - c_L) + c_L\}] / 2$$

or,
$$p_1 = 4 + 2p_j - [2 + p_j - \{\theta(c_H - c_L) + c_L\}] / 2$$

or,
$$p_1^* = 2 + p_j + \theta(c_H - c_L) - c_L / 2$$
(5)

Similarly,

$$\mathbf{p}_2 = 2 + \mathbf{p}_j - \mathbf{q}_2$$

or,
$$p_2 = 2 + p_j - (1 + p_j / 2)$$
 [Since $q_2 = (1 + p_j) / 2$]

or, p₂^{*}=(3 + p_j)/2 xperts Mind⁽⁶⁾

Equations: (5) and (6) are the Bayesian Nash Equilibrium (BNE) of the game

One important factor has to be noted in this regard. Since 1^{st} firm's equilibrium price (p_1^*) depends upon the stochastic model, hence as the probability of higher marginal cost (MC) increases, so does the value of p_1^* .

In other words, $p_1^* = f(p_j, \omega)$ with $f'(p_j)$, $f'(\omega) > 0$.

[Where $\omega = \theta(c_H - c_L) - c_L$]

Similarly, $p_2^* = g(p_j)$ with g' > 0.

The **major** difference between the two equilibrium prices of two firms is that the equilibrium price of the 1^{st} firm *not* only depends upon 'p_j' (price charged by firm j) but also on the expectations of the difference between high and low marginal costs.

On the other hand, as the MC of the 2^{nd} firm is already known, hence the equilibrium price of the 2^{nd} firm will depend *only* on 'p_j' (price charged by firm j).

Answer# 4:

Let $c_1 = MC_1 = Marginal \text{ cost for firm: } 1 = \theta. c_H + (1 - \theta). c_L$

The equilibrium condition $MR_1 = MC_1$ gives us:

$$2 + p_j - 2q_1 = c_1$$

or, dC₁ / dq₁ = 2 + p_j - 2q₁
or, dC₁ = (2 + p_j - 2q₁) dq₁

Now integrating both sides we get:

 $\int dC_{1} = 2 \int dq_{1} + p_{j} \int dq_{1} - q_{1}^{2} + F$

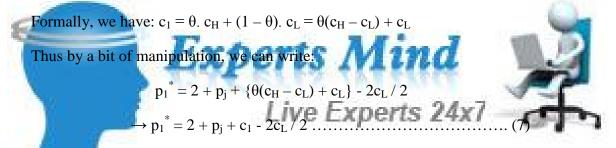
or, $C_1=2q_1+p_j\,{q_1}$ - ${q_1}^2+F=Cost$ of production for 1^{st} firm

where F = Fixed cost (Constant of integration).

Now we know that $p_1^* = 2 + p_j + \theta(c_H - c_L) - c_L / 2$ and

 $p_2^* = (3 + p_j) / 2$ (From equations: 5 and 6).

Hence when firm: 1 has a high c_H , given the value of p_j , the equilibrium price (p_1^*) will increase which in turn will effect on its output (q_1) .



Equation: (7) shows the relationship between BNE price and the marginal cost for firm: 1.

Answer# 5:

We already have as per equation (7):

 $p_1^* = 2 + p_j + c_1 - 2c_L / 2$

Now according to the problem, if firm: 1 does not know about 'c₁' and 'c_L'; then we may hold

 $c_1 = c_L = 0$ which simplifies equation: (8) into: $p_1^* = (2 + p_j) / 2$.

In this case, firm: 1 has to set its equilibrium price based on the price charged by firm $j(p_j)$.

Thus in this method (equilibrium concept), the 1st firm would be able to solve the game for Bayesian Nash Equilibrium (BNE).

