Problem 2. Fix a DFA $M = (Q, \Sigma, \delta, q_0, F)$. For any two states $q, q' \in Q$, let us say that q and q' are *equivalent*, written $q \sim q'$, if, for all $w \in \Sigma^*$ we have that $\delta^*(q, w) \in F \Leftrightarrow \delta^*(q', w) \in F$. Here δ^* is the extension of δ to Σ^* defined by $\delta^*(q, \varepsilon) = q$ and $\delta^*(q, ax) = \delta^*(\delta(q, a), x)$.

(a) Prove that $\sim$ is an equivalence relation.

(b) Suppose that $q \sim q'$ for distinct q, q'. Describe, first in plain English and then in precise mathematical terms, how to construct a smaller (=fewer state) DFA M' that accepts the same language as M.

Problem 4. Give DFAs for the following languages. Assume an alphabet that includes all and only the mentioned symbols. Make your DFA as small as possible.

(a) The set of all strings that have `abba` as a substring.

(b) The set of all strings that do not have `abba` as a substring.

Answer 2a. DFA $M=(Q,\Sigma,\delta, q_0,F)$.for any two states $q,q' \in Q$,

$q \approx q'$ $\forall w \in \Sigma^*$: $\delta^*(q, w) \Leftrightarrow \delta^*(q', w)$

where $\delta^*(q,e)=q$ and $\delta^*(q,ax)= \delta^*(\delta(q,a),x)$

A relation is $\approx$ is equivalence if

(i) It is reflexive i.e. $x \equiv x$

(ii) It is symmetric ie $x \equiv y \Rightarrow y \equiv x$

(iii) It is transitive ie $x \equiv y \wedge y \equiv z \Rightarrow x \equiv z$

$\forall w \in \Sigma$ $\delta(q, w) \equiv \delta(q', w)$

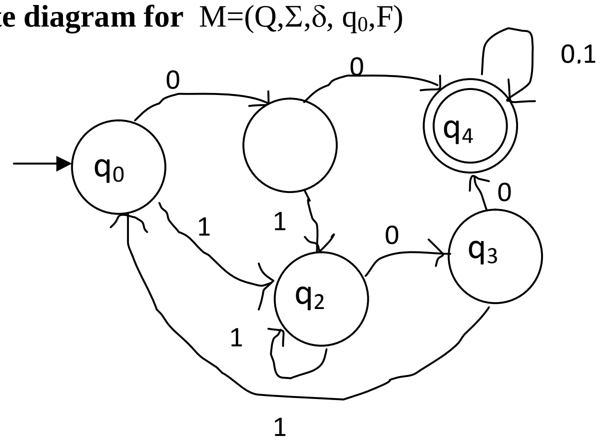
$\Leftrightarrow \forall a \in \Sigma$ $\forall x \in \Sigma^*$: $\delta^*(\delta(q, a),x) \in F \equiv \delta^*(\delta(q', a),x) \in F$

$\Leftrightarrow \forall a \in \Sigma$ $\forall x \in \Sigma^*$: $\delta^*(q,ax) \in F \equiv \delta^*(q',ax) \in F$

$\Leftrightarrow$ $\forall w' \in \Sigma^*$: $\delta^*(q,w') \in F \equiv \delta^*(q',w') \in F$

$\Leftrightarrow q \equiv q'$

2b. state diagram for M=(Q,Σ,δ, q_0,F)



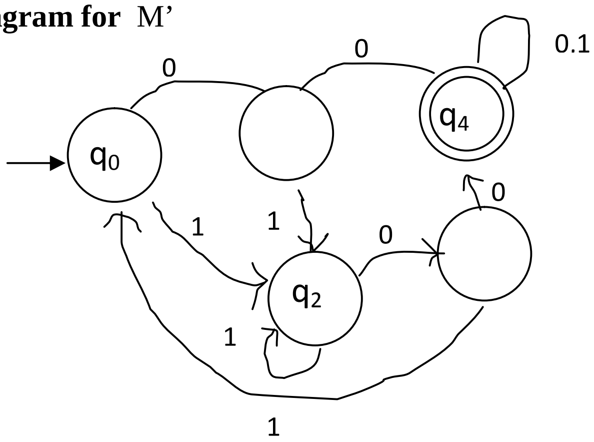
$[q_0] = \{q_0, q_2\}$

$[q_1] = \{q_1, q_3\}$

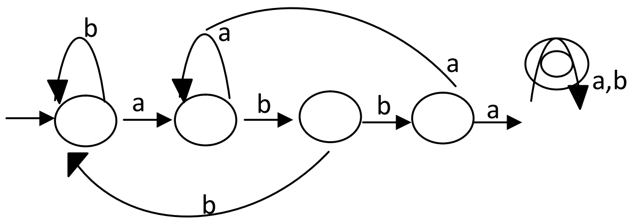
$[q_4] = \{q_4\}$

	q_0	q_1	q_2	q_3	q_4
0	-	F	-	F	F
1	-	-	-	-	F
00	F	F	F	F	F
01	-	F	-	F	F
10	-	-	-	-	F
11	-	-	-	-	F
000	F	F	F	F	F
001	F	F	F	F	F
010	-	F	-	F	F
011	-	F	-	F	F
100	F	F	F	F	F
101	-	-	-	-	F

state diagram for M’



4(a) set of all strings that have abba as substring.



4(b) set of all strings that do not have abba as substring.