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Question 1

Integrate the following functions, or solve the given ODE.

(a) $f(x) = \cos(\ln(x))$

(b) $\frac{dy}{dx} = \frac{1}{3+5\cos(x)}$ (Hint: use t substitution.)

(c) $\frac{dy}{dx} = \sqrt{x^2 - 8x - 9}$ (Hint: think of using an hyperbolic substitution instead of trigonometric.)

Solution:

(a) f(x)= Cos (ln(x))

$I = \int Cos(\ln(x))dx$

Put ln(x) = t

$X = e^t$

Differentiating $dx = e^t dt$

Then

$I = \int cos t e^t dt$

Integrating by parts we get

$\int udv = uv - \int vdu$ $\quad dv = e^t dt \quad u = cost$

$V = e^t \quad du = -sint\ dt$

$I = e^t cost - \int e^t(-sint)dt$

$I = e^t cost + \int e^t(sint)dt$

Further integrating by parts we get

$dv = e^t dt \quad u = sint$

$V = e^t \quad du = Cost\ dt$

$I = e^t cost + e^t sint - \int e^t(cost)dt$

$I = e^t cost + e^t sint - I$

$2I = e^t cost + e^t sint$

$I = \frac{1}{2}[e^t cost + e^t sint]$

$I = \frac{1}{2}[e^{\ln(x)} cos(\ln(x)) + e^{\ln(x)} sin(\ln(x))]$

$$I = \frac{1}{2}[x\cos(\ln(x)) + x\sin(\ln(x))]$$

$$I = \frac{x}{2}[\cos(\ln(x)) + \sin(\ln(x))] + C$$

(b) $\frac{dy}{dx} = \frac{1}{3 + 5\cos(x)}$

$$y = \int \frac{dx}{3 + 5\cos(x)}$$

$$y = \int \frac{dx}{3 + 5\left[\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right]}$$

$$y = \int \frac{\left[1 + \tan^2\frac{x}{2}\right]dx}{3 + 3\tan^2\frac{x}{2} + 5 - 5\tan^2\frac{x}{2}}$$

$$y = \int \frac{\mathrm{Sec}^2\frac{x}{2}dx}{8 - 2\tan^2\frac{x}{2}}$$

$$y = \int \frac{\mathrm{Sec}^2\frac{x}{2}dx}{2\left[4 - \tan^2\frac{x}{2}\right]}$$

Put $\tan\frac{x}{2} = t$; differentiating we get

$$\frac{1}{2}\mathrm{Sec}^2\frac{x}{2}dx = dt$$

$$y = \int \frac{dt}{4 - t^2}$$

$$y = \frac{1}{4}\int \frac{2 - t + 2 + t}{(2 - t)(2 + t)}dt = \frac{1}{4}\left[\int \frac{dt}{2 - t} + \int \frac{dt}{2 + t}\right] = \frac{1}{4}\ln\frac{2 + t}{2 - t} = \frac{1}{4}\ln\frac{2 + \tan\frac{x}{2}}{2 - \tan\frac{x}{2}} + c$$

(c) $\frac{dy}{dx} = \sqrt{x^2 - 8x - 9}$

$$\frac{dy}{dx} = \sqrt{x^2 - 8x + 16 - 16 - 9}$$

$$\frac{dy}{dx} = \sqrt{(x - 4)^2 - 25}$$

$$y = \int \sqrt{(x - 4)^2 - 25}\, dx$$

Put x-4= 5 coshu

Differentiating dx= 5 sinhu du

$$y = \int \sqrt{25\cosh^2 u - 25}\, 5\sinh u\, du = \int 25\sinh^2 u\, du$$

$$y=\int 25\left[\frac{e^{u}-e^{-u}}{2}\right]^{2}du=\frac{25}{4}\int[e^{2u}+e^{-2u}-2]du=\frac{25}{8}\left[e^{2u}-e^{-2u}-4u\right]$$

$$y=\frac{25}{8}\left[\left(e^{u}-e^{-u}\right)\left(e^{u}+e^{-u}\right)-4u\right]$$

$$y=\frac{25}{8}[4\sinh u\cosh u-4u]=\frac{25}{2}\left[\frac{x-4}{5}\frac{\sqrt{x^{2}-8x-9}}{5}-\cosh^{-1}\frac{x-4}{5}\right]+c$$

Question 2

(a) Find $\int u\cosh(u)du$.

(b) Solve the ODE $\frac{dy}{dx}+\frac{y}{x+2}=\left(x^{2}+4\right)\cosh\left(x^{2}+4\right)$.

(c) A child's toy is to be made from the region contained between the curve $y=x\cosh(x), -\pi\le x\le\pi$ and the *x* axis, mounted upon a stick so it can revolve in the breeze. The region will be cut out from a thin sheet of density ρ.

1) Use MATLAB to generate a plot of the region concerned.
2) Find the mass of the blade.
3) State where you expect the centre of mass of this shape to be, using your understanding of centre of mass. Confirm this by making a calculation for the *x* coordinate of the centre of mass only.

Solution:

(a)

$I=\int u\cosh u\,du$ $\quad$ $dv=\cosh u\,du$ $\quad$ $t=u$

$V=\sinh u$ $\quad$ $dt=du$

$$\int t\,dv=tv-\int v\,dt$$

$$=u\sinh u-\int\sinh u\,du$$

$$=u\sinh u-\cosh u+c$$

(b) $\frac{dy}{dx}+\frac{y}{x+2}=(x^{2}+4)\mathrm{Cosh}(x^{2}+4)$

This is a linear differential equation.

$P=\frac{1}{x+2}$ and $Q=(x^{2}+4)\mathrm{Cosh}(x^{2}+4)$

Integrating factor is

$$e^{\int p\,dx}=e^{\int\frac{dx}{x+2}}=e^{\ln(x+2)}=(x+2)$$

Multiplying the given ODE by the integrating factor we get

$$(x+2)\frac{dy}{dx}+y=(x+2)(x^{2}+4)\mathrm{Cosh}(x^{2}+4)$$

$$\frac{d(x+2)y}{dx}=(x+2)(x^{2}+4)\mathrm{Cosh}(x^{2}+4)$$

Integrating on both sides

$Y(x+2) = \int (x+2)(x^2+4)\text{Cosh}(x^2+4)dx = \int x(x^2+4)\text{Cosh}(x^2+4)dx + 2\int (x^2+4)\text{Cosh}(x^2+4)dx = I_1 + I_2$

In the first integral Put $(x^2+4) = t$ differentiating we get $2x\,dx = dt$ then it gets transformed to

$I_1 = \frac{1}{2}\int t\cosh t\,dt = \frac{1}{2}\{t\sinh t - \cosh t\} + C = \frac{1}{2}\{(x^2+4)\text{Sinh}(x^2+4) - \cosh(x^2+4)\} + C$

I_2 is not a standard integral. This has to be done by series method.

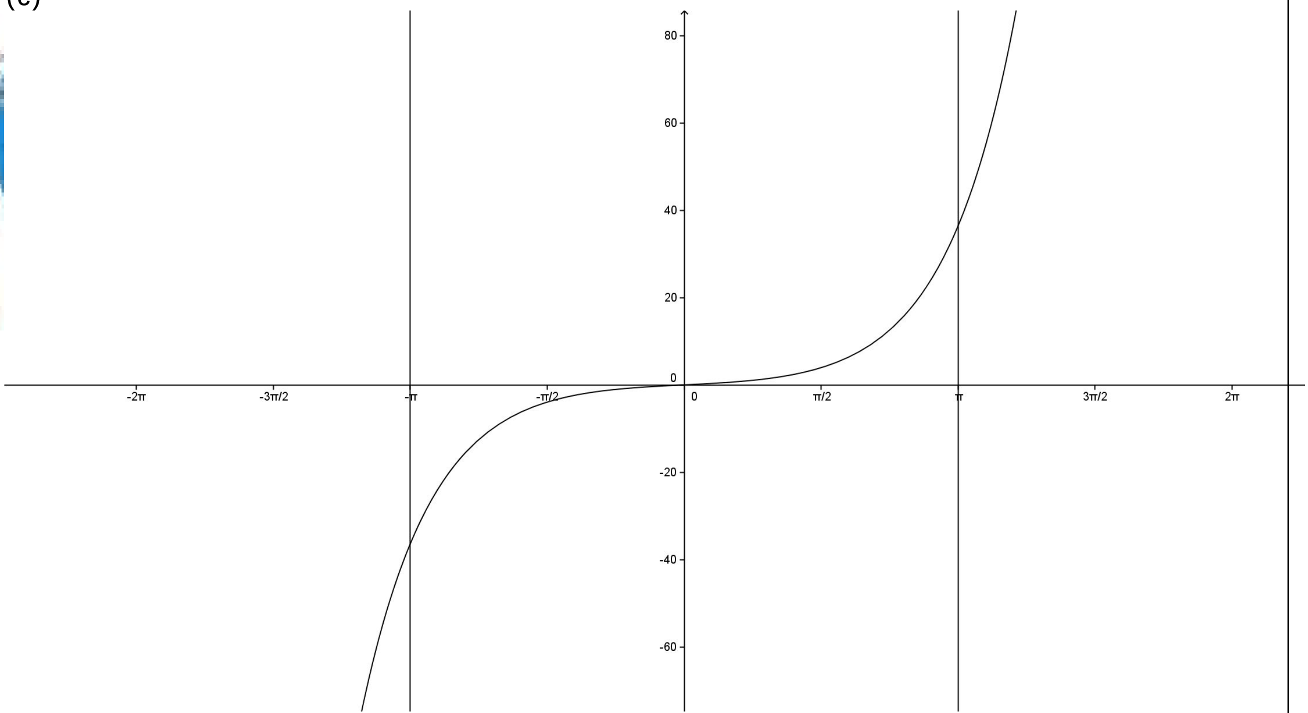
We know that $\text{Coshu} = \frac{e^u - e^{-u}}{2} = 1 + \frac{u^2}{2*} + \frac{u^4}{4*} \ldots$, where * stands for factorial of a number.

In our case $u = x^2+4$, replacing we get $= \text{Cosh}(x^2+4)\, 1 + \frac{(x^2+4)^2}{2*} + \frac{(x^2+4)^4}{4*} \ldots$

$I_2 = \int 1 + \frac{(x^2+4)^3}{2*} + \frac{(x^2+4)^5}{4*} \ldots dx$ expanding and integrating we get

$I_2 = x + \frac{1}{2}\left[\frac{x^7}{7} + \frac{12x^5}{5} + 24x^2 + 64x\right] \ldots\ldots$

(c)



Area under region

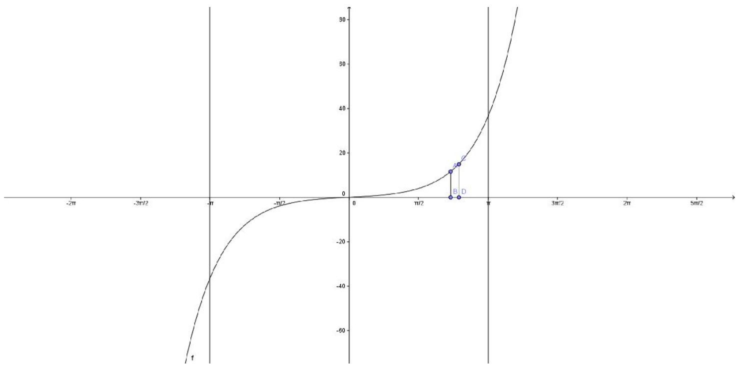
$\int_{-\pi}^{\pi} x\cosh x\,dx = 2\int_{0}^{\pi} x\cosh x\,dx$ due to its symmetry about y- axis.

$= 2[x\sinh x - \cosh x]_0^{\pi} = 2[\pi\sinh\pi - \cosh\pi + 1]$

If the area density of the sheet is ρ, then the mass of the blade is $2\rho[\pi\sinh\pi - \cosh\pi + 1]$.

Center of mass

Consider a thin strip of thickness dx at a distance of x from the origin as shown in the figure



Mass of the strip is $\rho x\cosh(x)dx$

Then the center of mass= $\dfrac{\int_{-\pi}^{\pi} x^2 \cosh x\, dx}{M} = \dfrac{\left[x^2 \mathrm{Sinh}x - 2x\cosh x - 2\sinh x\right]_{-\pi}^{\pi}}{M}$ (On integrating by Parts successively)

Center of mass = 0 as the numerator vanishes. The center of mass lies at the origin which is true from its symmetry.

Question 3

(a) By reducing the second order ODE to a system of two first order ODE's, the general solution to $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$. Find the eigenvectors and eigenvalues by hand and check them using MATLAB.

(b) Find any solution to the non-homogeneous ODE $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = e^x + 4x$

Solution: (a) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4 = 0$

Auxiliary equation: $m^2 - 4m + 4 = 0$; $(m-2)^2=0$; m=2 the roots are real and equal.

Hence $y=(Ax+B)e^{2x}$

Eigen vectors:

Eigen values:

(b) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4 = e^x + 4x$

Auxiliary equation: $m^2 - 4m + 4 = 0$; $(m-2)^2=0$; m=2 the roots are real and equal.

Hence

General solution: $y=(Ax+B)e^{2x}$

Particular Integral:

$$PI = \frac{e^x + 4x}{D^2 - 4D + 4} = \frac{e^x}{D^2 - 4D + 4} + \frac{4x}{D^2 - 4D + 4} = \frac{e^x}{(1)^2 - 4(1) + 4} + \frac{4x}{4(1 - D + D^2/4)}$$

$= e^x + (1-D+D^2/4)^{-1}(x) = e^x + (x+1)$

Hence Y= GS+PI= $y=(Ax+B)e^{2x} + e^x + (x+1)$

(c) Use the initial conditions $y(0)=3, \left.\frac{dy}{dx}\right|_{x=0}=6$ to find the unknown constant hence find the solution satisfying these conditions.

Y(0) = 3; hence 3=B+2; B= 1
Y'= (Ax+B) 2 e^{2x} +A e^{2x} +e^{x} +1
At x= 0 Y'=6
6= 2B +A +2; A= 2
Substituting the values of A and B in Y= (Ax +B)e^{2x} + e^{x} + (x+1); we get
Y= (2x +1)e^{2x} + e^{x} + (x+1)

Question 4

(a) Find the integral of $\left(\cos^{-1}(x)\right)^2$, using integration by parts.

(b) Consider the region between the curve $y=\cos(x), \frac{-\pi}{2} \le x \le \frac{\pi}{2}$ and the x a

1) Find the area of this region.
2) Assuming a density of ρ, find the centre of mass. You may assume th the x coordinate of the centre of mass occurs on the vertical axis.

Solution:

(a) $I = \int \left[\cos^{-1}x\right]^2 dx$ Put x= Cost; differentiating we get dx= sint dt

$I = \int t^2(\text{sint})dt$ dv= -sint dt u= t^2

V= cost du=2tdt

$\int udv = uv - \int vdu$

$I = t^2\text{cost} - \int 2t\text{cost}dt$

$I = t^2\text{cost} - I_1$ where

$I_1 = \int 2t\text{cost}dt$ dv= cost dt u=2t

V= sint du= 2dt

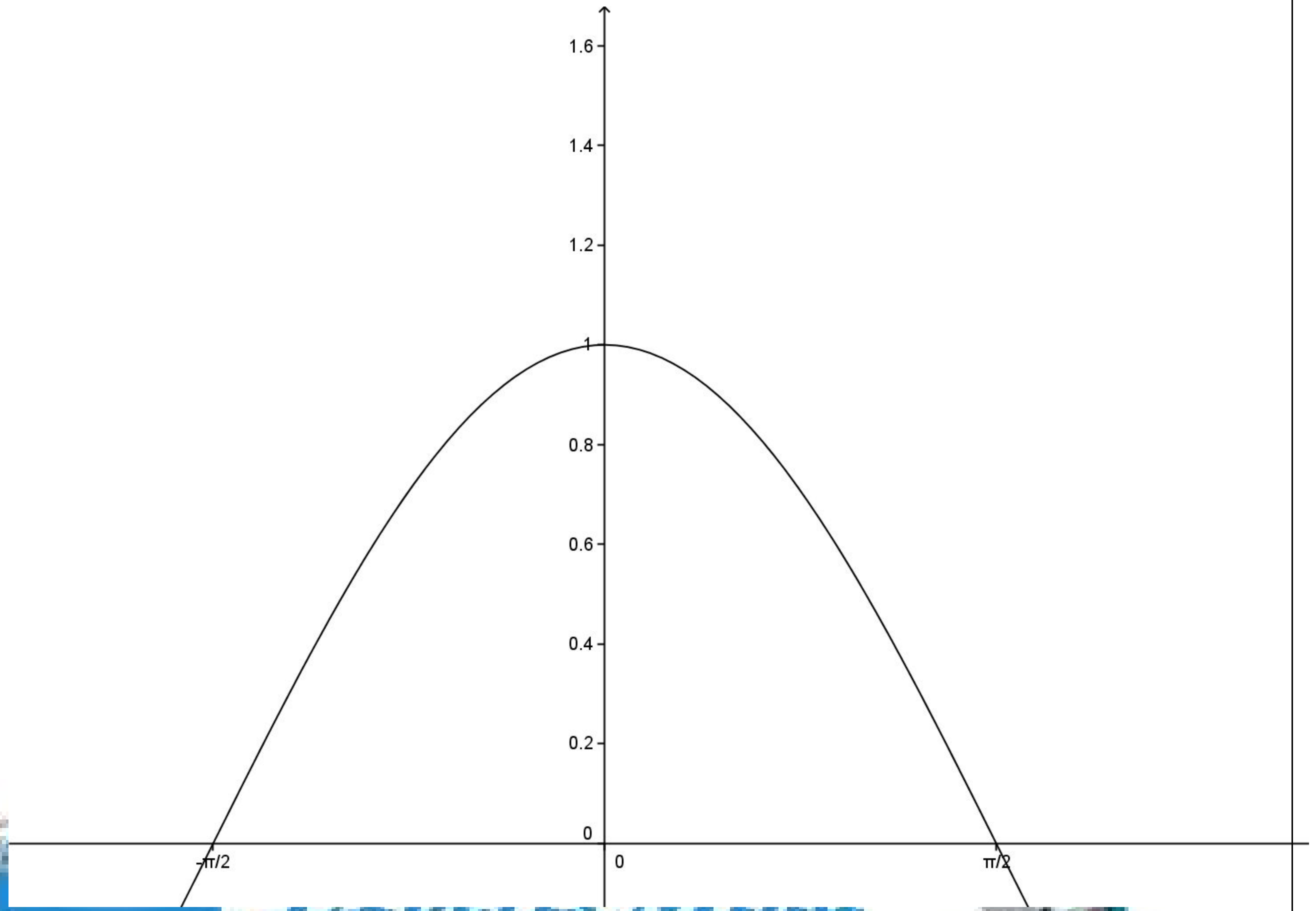
$I_1 = 2t\text{sint} - \int 2\text{sint}dt$

$I_1 = 2t\text{sint} + 2\text{cost}$

I= t^2cost-2tsint-2cost+c { x= Cost; $\sqrt{1-x^2}$ = sint ; t= $\cos^{-1}x$ }

I= $x\left[\cos^{-1}x\right]^2 - 2\cos^{-1}x\sqrt{1-x^2} - 2x + C$

(b) Trace of the given curve

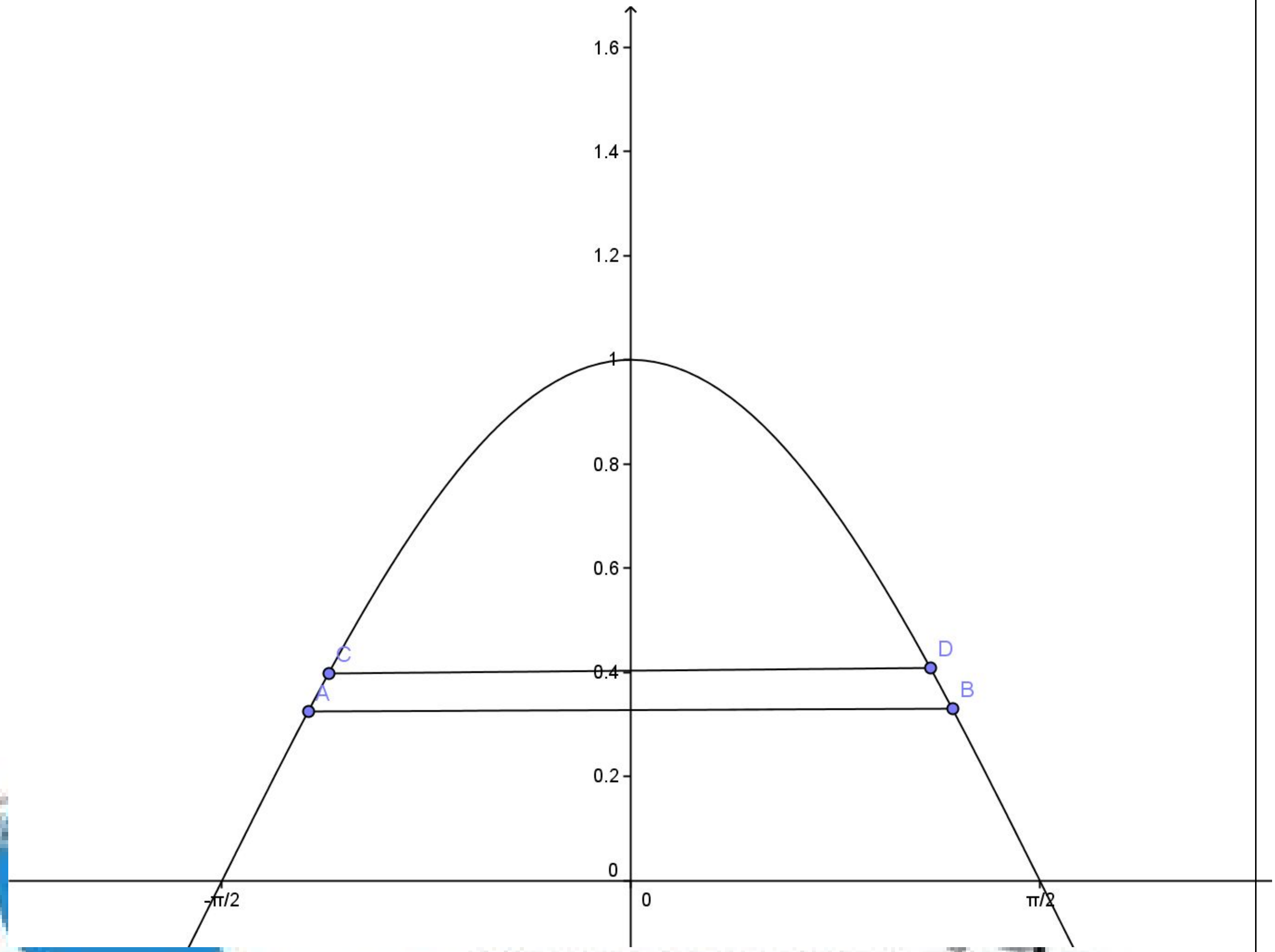


Curve is symmetric about the Y axis

Hence $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2\int_{0}^{\frac{\pi}{2}} \cos x dx = 2[\sin x]_{0}^{\frac{\pi}{2}} = 2[1-0] = 2$

(b) Consider a small strip of thickness dx at a distance of x from the origin as shown in the figure

To find the position on the Y axis,



Consider a thin horizontal strip of thickness dy at a distance of y from the origin; the area of the strip is $2\cos^{-1}y\,dy$. Hence the mass of the strip is $\rho 2\cos^{-1}y\,dy$.

Center of mass $= \dfrac{\int_0^1 \rho 2y\cos^{-1}y\,dy}{M} = \dfrac{\int_0^1 \rho 2y\cos^{-1}y\,dy}{2\rho} = \int_0^1 y\cos^{-1}y\,dy =$. Integrating by parts we get

After substituting y = cost we get

$$\int_0^1 y\cos^{-1}y\,dy = \int_0^{\frac{\pi}{2}} \cos t \sin t\; t\,dt = \frac{1}{2}\int_0^{\frac{\pi}{2}} t\sin 2t\,dt = \frac{1}{8}\left[-2t\cos 2t + \sin 2t\right]_0^{\frac{\pi}{2}} = \frac{\pi}{8}$$

Hence the center of mass of the strip is $(0, \frac{\pi}{8})$

Microsoft Equation 3.0