Exercise 3 - 40%

Two firms, $i = 1, 2$, sell differentiated goods. The firms set prices p_1 and p_2 simultaneously. We assume that prices have to be non-negative, $p_1, p_2 \geq 0$. The demand function facing firm i is given by

$$q_i(p_i, p_j) = 2 - p_i + p_j$$

(do not worry about negative demand when doing this exercise - in equilibrium demand will be positive).

Firm 1 can have low marginal cost, c_L, or high marginal cost, c_H, where $0 < c_L < c_H < 2$. Firm 1 knows whether its marginal cost is low or high. Firm 2 only knows that the probability that Firm 1 has high marginal cost is θ, where $0 < \theta < 1$. Firm 2's marginal cost is $c = 1$ and this is known by both firms. (All this is common knowledge for the firms).

1. Formulate the situation as a static game of incomplete information, i.e., specify action spaces, type spaces, beliefs, and payoff functions for the two firms.
2. Find the best response functions.
3. Find the Bayesian Nash equilibrium of the game (i.e., find the equilibrium prices).
4. Compare the BNE prices if Firm 1's realized cost is high with the equilibrium prices in the game where it is common knowledge that Firm 1 has high marginal cost (c_H).
5. Suppose now that Firm 1 does not observe its own marginal cost before it sets its price. I.e., Firm 1 only knows that the probability that it has high marginal cost (c_H) is θ. Everything else is as in the original set-up. Which equilibrium concept would you use to solve this game? Explain.

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Answer# 1:

Given the demand function: $q_i\ (p_i, p_j) = 2 - p_i + p_j$ ($i = 1, 2$ and $i \neq j$)

or, $p_i = 2 + p_j - q_i \rightarrow p_1 = 2 + p_j - q_1$ and $p_2 = 2 + p_j - q_2$

$\rightarrow R_i$ = Revenue of firm i = $p_i\, q_i = 2q_i + p_j\, q_i - q_i^2$

Thus $R_1 = 2q_1 + p_j\, q_1 - q_1^2$ = Revenue of 1st firm,

and $R_2 = 2q_2 + p_j\, q_2 - q_2^2$ = Revenue of 2nd firm.

According to the problem, firm: 2 only know that the probability of having high MC of firm: 1 is 'θ'.

So formally, $p(c_H)$ = Probability of having high MC = θ and

$p(c_L)$ = Probability of having low MC = $1 - \theta$ where $\theta < 1$.

Hence we can express MC_1 = Marginal cost for firm: 1 = $p(c_H).\ c_H + p(c_L).\ c_L$

$$= \theta.\ c_H + (1 - \theta).\ c_L$$

Now MR_1 = Marginal revenue of 1st firm = $2 + p_j - 2q_1$

So the equilibrium condition $MR_1 = MC_1$ gives us:

$$2 + p_j - 2q_1 = \theta.\ c_H + (1 - \theta).\ c_L$$

$$\text{or, } 2 + p_j - 2q_1 = \theta(c_H - c_L) + c_L$$

$$\text{or, } 2q_1 = 2 + p_j - [\theta(c_H - c_L) + c_L]$$

$$\text{or, } q_1 = 2 + p_j - [\theta(c_H - c_L) + c_L] / 2 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

Similarly, MR_2 = Marginal revenue of 2nd firm = $2 + p_j - 2q_2$

Given, MC_2 = Marginal cost of firm: 2 = c = 1.

Hence the equilibrium condition $MR_2 = MC_2$ gives us:

$$2 + p_j - 2q_2 = 1$$

$$\text{or, } 2q_2 = 1 + p_j$$

$$\text{or, } q_2 = 1 + p_j / 2 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2)$$

Equations: (1) and (2) show the situations of static game of incomplete information.

The payoff of 1st firm: $S_1(\theta) = 0$ and the payoff of 2nd firm: $S_2(\theta) = 1$.

<u>Answer# 2</u>:

According to equation: (1) we have:

$q_1 = 2 + p_j - [\theta(c_H - c_L) + c_L] / 2$

or, $q_1 = [(1 + p_j) + 1 - \theta(c_H - c_L) - c_L] / 2$

or, $q_1 = [(1 + p_j) / 2] + [1 - \theta(c_H - c_L) - c_L / 2]$ …………………………….. (3)

We know that: $(1 + p_j) / 2 = q_2$. So substituting the value of q_2 in equation: (3) we get:

$q_1^* = q_2^* + [1 - \theta(c_H - c_L) - c_L / 2] = q_2^* + \rho(c_H, c_L)$ ………………………… (4)

where q_1^* = Equilibrium demand function for 1st firm

= *Response function for 1st firm* as a function of q_2 with $\rho'(c_H) > 0$, $\rho'(c_L) < 0$.

And $q_2^* = q_1^* - \rho(c_H, c_L) \rightarrow$ *Response function for 2nd firm* as a function of q_1 with $\rho'(c_H) > 0$, $\rho'(c_L) < 0$.

According to the problem, we have: $0 < c_L < c_H < 2$ and $0 < \theta < 1$.

Now for the sake of simplicity, let us take an example that $\theta = 0.5$, $c_L = 0.5$ and $c_H = 1$.

Thus putting the values of c_L, c_H, and θ in equation: (4) we get:

$q_1^* = q_2^* + [(1 - 0.25 - 0.5) / 2] = q_2^* + 0.125$ – which is a **simplified response**

function for firm: 1 with *known* values of the variables.

In a similar fashion, we can derive the simplified response function for the 2nd firm with known values of the variables c_L, c_H, and θ.

However the two equilibrium values of q_1 and q_2 will become *equal* when

$1 - \theta(c_H - c_L) - c_L / 2 = 0$

or, $1 - \theta(c_H - c_L) - c_L = 0$

or, $\theta(c_H - c_L) = 1 - c_L \rightarrow \boldsymbol{\theta = (1 - c_L) / (c_H - c_L)}$

In other words, $q_1^* = q_2^*$ when $\theta = (1 - c_L) / (c_H – c_L)$ – which contemplates the situation of a static game with complete information about the market (Pure Strategy Nash Equilibrium solution).

<u>Answer# 3</u>:

From our previous discussion, we have:

$$p_1 = 2 + p_j – q_1 \text{ and } p_2 = 2 + p_j – q_2$$

Therefore, $p_1 = 2 + p_j - [2 + p_j – \{\theta(c_H – c_L) + c_L\}] / 2$

as $q_1 = [2 + p_j – \{\theta(c_H – c_L) + c_L\}] / 2$

or, $p_1 = 4 + 2p_j – [2 + p_j – \{\theta(c_H – c_L) + c_L\}] / 2$

or, $p_1^* = 2 + p_j + \theta(c_H – c_L) - c_L / 2$ …………………………… (5)

Similarly,

$$p_2 = 2 + p_j – q_2$$

or, $p_2 = 2 + p_j – (1 + p_j / 2)$ [Since $q_2 = (1 + p_j) / 2$]

or, $p_2^* = (3 + p_j) / 2$ ……………………………………………… (6)

Equations: (5) and (6) are the Bayesian Nash Equilibrium (BNE) of the game.

One important factor has to be noted in this regard. Since 1st firm's equilibrium price (p_1^*) depends upon the stochastic model, hence as the probability of higher marginal cost (MC) increases, so does the value of p_1^*.

In other words, $p_1^* = f(p_j, \omega)$ with $f'(p_j), f'(\omega) > 0$.

[Where $\omega = \theta(c_H – c_L) - c_L$]

Similarly, $p_2^* = g(p_j)$ with $g' > 0$.

The **major** difference between the two equilibrium prices of two firms is that the equilibrium price of the 1st firm *not* only depends upon 'p_j' (price charged by firm j) but also on the expectations of the difference between high and low marginal costs.

On the other hand, as the MC of the 2nd firm is already known, hence the equilibrium price of the 2nd firm will depend *only* on 'p_j' (price charged by firm j).

<u>Answer# 4</u>:

Let $c_1 = MC_1$ = Marginal cost for firm: $1 = \theta. c_H + (1 – \theta). c_L$

The equilibrium condition $MR_1 = MC_1$ gives us:

$$2 + p_j - 2q_1 = c_1$$

$$\text{or, } dC_1 / dq_1 = 2 + p_j - 2q_1$$

$$\text{or, } dC_1 = (2 + p_j - 2q_1)\, dq_1$$

Now integrating both sides we get:

$$\int dC_1 = 2 \int dq_1 + p_j \int dq_1 - q_1^2 + F$$

$$\text{or, } C_1 = 2q_1 + p_j\, q_1 - q_1^2 + F = \text{Cost of production for 1}^{st} \text{ firm}$$

where F = Fixed cost (Constant of integration).

Now we know that $p_1^* = 2 + p_j + \theta(c_H - c_L) - c_L / 2$ and

$p_2^* = (3 + p_j) / 2$ (From equations: 5 and 6).

Hence when firm: 1 has a high c_H, given the value of p_j, the equilibrium price (p_1^*) will increase which in turn will effect on its output (q_1).

Formally, we have: $c_1 = \theta.\, c_H + (1 - \theta).\, c_L = \theta(c_H - c_L) + c_L$

Thus by a bit of manipulation, we can write:

$$p_1^* = 2 + p_j + \{\theta(c_H - c_L) + c_L\} - 2c_L / 2$$

$$\rightarrow p_1^* = 2 + p_j + c_1 - 2c_L / 2 \text{ ……………………………… (7)}$$

Equation: (7) shows the relationship between BNE price and the marginal cost for firm: 1.

Answer# 5:

We already have as per equation (7):

$$p_1^* = 2 + p_j + c_1 - 2c_L / 2$$

Now according to the problem, if firm: 1 does not know about 'c_1' and 'c_L'; then we may hold

$c_1 = c_L = 0$ which simplifies equation: (8) into: $p_1^* = (2 + p_j) / 2$.

In this case, firm: 1 has to set its equilibrium price based on the price charged by firm j (p_j).

Thus in this method (equilibrium concept), the 1st firm would be able to solve the game for Bayesian Nash Equilibrium (BNE).